Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **For examination from 2020**

SPECIMEN PAPER **3 hours**

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You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 120.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

Section A: Probability (60 marks)

1 The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = at\left(t + \frac{1}{t}\right)^3,$$

where a is a constant.

(a) Find, in either order, the value of a and the set of values that X can take. [4]

(b) Find the value of $E(X)$. [2]

2 **(a)** The probability that a shopper obtains a parking space on the river embankment on any given Saturday morning is 0.2. Using a suitable normal approximation, find the probability that, over a period of 100 Saturday mornings, the shopper finds a parking space at least 15 times. Justify the use of the normal approximation in this case. [5]

(b) The number of parking tickets that a traffic warden issues on the river embankment during the course of a week has a Poisson distribution with mean 36. The probability that the traffic warden issues more than N parking tickets is less than 0.05. Using a suitable normal approximation, find the least possible value of N. [5]

3 Small amounts of a potentially hazardous chemical are discharged into a river from a nearby industrial site. A random sample of size 6 was taken from the river and the concentration of the chemical present in each item was measured in grams per litre. The results are shown below.

1.64 1.53 1.78 1.60 1.73 1.77

(a) Assuming that the sample was taken from a normal distribution with known variance 0.01, construct a 99% confidence interval for the mean concentration of the chemical present in the river. [4]

(b) If instead the sample was taken from a normal distribution, but with unknown variance, construct a revised 99% confidence interval for the mean concentration of the chemical present in the river. [5]

(c) If the mean concentration of the chemical in the river exceeds 1.8 grams per litre, then remedial action needs to be taken. Comment briefly on the need for remedial action in the light of the results in parts **(a)** and **(b)**. [1]

4 The independent random variables X and Y have normal distributions where $X \sim \mathrm{N}(\mu,\ \sigma^2)$ and $Y \sim \mathrm{N}(3\mu,\ 4\sigma^2)$. Two random samples each of size n are taken, one from each of these normal populations.

(a) Show that $a\overline{X} + b\overline{Y}$ is an unbiased estimator of μ provided that $a + 3b = 1$, where a and b are constants and $\overline{X}$ and $\overline{Y}$ are the respective sample means. [3]

In the remainder of the question assume that $a\overline{X} + b\overline{Y}$ is an unbiased estimator of μ.

(b) Show that $\mathrm{Var}\left(a\overline{X} + b\overline{Y}\right)$ can be written as $\dfrac{\sigma^2}{n}(1 - 6b + 13b^2)$. [3]

(c) The value of the constant b can be varied. Find the value of b that gives the minimum of $\mathrm{Var}\left(a\overline{X} + b\overline{Y}\right)$, and hence find the minimum of $\mathrm{Var}\left(a\overline{X} + b\overline{Y}\right)$ in terms of σ and n. [3]

5 The random variable X has probability density function $\mathrm{f}(x)$, where

$$\mathrm{f}(x) = \begin{cases} ke^{-kx} & x \geqslant 0, \\ 0 & x < 0, \end{cases}$$

and k is a positive constant.

(a) Show that the moment generating function of X is $\mathrm{M}_X(t) = k(k - t)^{-1}$, $t < k$. [3]

(b) Use the moment generating function to find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [5]

(c) Show that the moment generating function of $-X$ is $k(k + t)^{-1}$. [1]

(d) X_1 and X_2 are two independent observations of X. Use the moment generating function of $X_1 - X_2$ to find the value of $\mathrm{E}[(X_1 - X_2)^2]$. [3]

6 The lengths of time, in years, that sales representatives for a certain company keep their company cars may be modelled by the distribution with probability density function f(x), where

$$f(x) = \begin{cases} \frac{4}{27}x^2(3-x) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Draw a sketch of this probability density function. [2]

(b) Calculate the mean and the mode of X. [6]

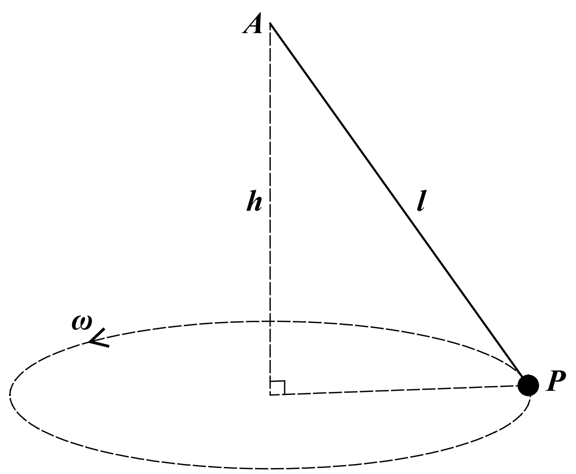
(c) Comment briefly on the values obtained in part **(b)** in relation to the sketch in part **(a)**. [1]

(d) Show that the lower quartile Q_1 of X satisfies the equation $Q_1^4 - 4Q_1^3 + 6.75 = 0$, and use an appropriate numerical method to find the value of Q_1 correct to 2 decimal places, showing full details of your method. [4]

Section B: Mechanics (60 marks)

7 A child of mass 20 kg slides down a rough slope of length 16 m against a constant frictional force F N. Starting with an initial speed of $2\,\text{m}\,\text{s}^{-1}$ at a point 8 m above the ground, she reaches the ground with a speed of $6\,\text{m}\,\text{s}^{-1}$. Find the value of F. [4]

8



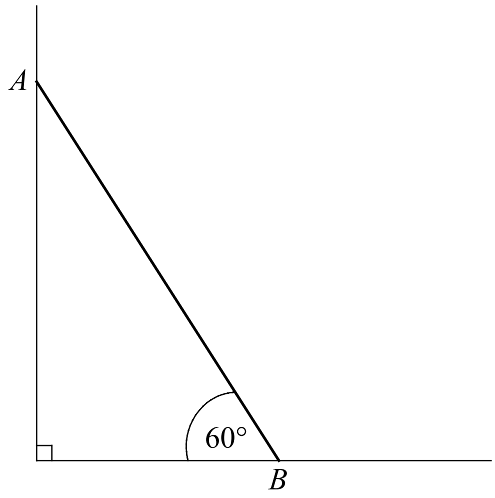
A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point A. The particle moves with constant angular speed ω in a horizontal circle whose centre is at a distance h vertically below A (see diagram).

(a) Find the tension in the string in terms of m, l and ω. [2]

(b) Show that $\omega^2 h = g$. [2]

(c) Deduce an expression in terms of g and h for the time taken for P to complete one full circle during its motion. [2]

9 The diagram shows a uniform rod AB of length 40 cm and mass 2 kg placed with the end A resting against a smooth vertical wall and the end B on rough horizontal ground. The angle between AB and the horizontal is 60°.



Given that the value of the coefficient of friction between the rod and the ground is 0.2, determine whether the rod slips. [6]

10 A cyclist and her bicycle have a combined mass of 90 kg and she is riding along a straight horizontal road. She is working at a constant power of 75 W. At time t seconds her speed is $v\,\mathrm{m\,s^{-1}}$ and the resistance to motion is kv N, where k is a constant.

(a) Given that the steady speed at which the cyclist can move is $10\,\mathrm{m\,s^{-1}}$, show that $k = \frac{3}{4}$. [1]

(b) Show that

$$\frac{25}{v} - \frac{v}{4} = 30\frac{\mathrm{d}v}{\mathrm{d}t}.$$ [2]

(c) Find the time taken for the cyclist to accelerate from a speed of $3\,\mathrm{m\,s^{-1}}$ to a speed of $7\,\mathrm{m\,s^{-1}}$. [5]

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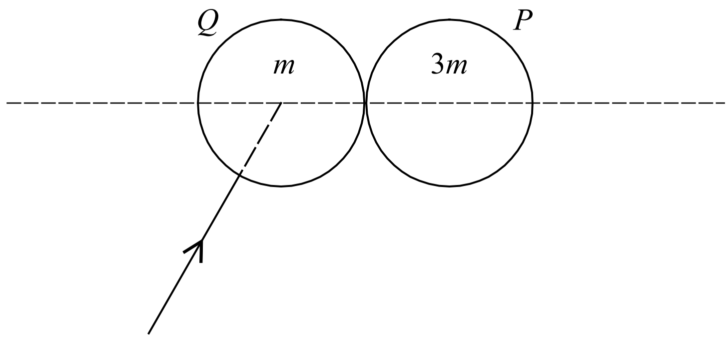

A smooth sphere P of mass $3m$ is at rest on a smooth horizontal table. A second smooth sphere Q of mass m and the same radius as P is moving along the table towards P and strikes it obliquely (see diagram). After the collision, the directions of motion of the two spheres are perpendicular.

(a) Find the coefficient of restitution. [5]

(b) Given that one-sixth of the original kinetic energy is lost as a result of the collision, find the angle between the initial direction of motion of Q and the line of centres. [7]

12 A particle is projected from the origin with speed $20\,\mathrm{m\,s^{-1}}$ at an angle α above the horizontal.

(a) Prove that the equation of its trajectory is

$$y = x\tan\alpha - \frac{x^2}{80}(l + \tan^2\alpha).$$ [4]

(b) Regarding the equation of the trajectory as a quadratic equation in $\tan\alpha$, show that $\tan\alpha$ has real values provided that

$$y \leqslant 20 - \frac{x^2}{80}.$$ [4]

(c) A plane is inclined at an angle of 30° to the horizontal. The line l, with equation $y = x\tan 30°$, is a line of greatest slope in the plane. The particle is projected from the origin with speed $20\,\mathrm{m\,s^{-1}}$ from a point on the plane, in the vertical plane containing l. By considering the intersection of l with the curve $y = 20 - \frac{x^2}{80}$, find the maximum range up this inclined plane. [4]

13 Two light strings, each of natural length l and modulus of elasticity $6mg$, are attached at their ends to a particle P of mass m. The other ends of the strings are attached to two fixed points A and B, which are at a distance $6l$ apart on a smooth horizontal table. Initially P is at rest at the mid-point of AB. The particle is now given a horizontal impulse in the direction perpendicular to AB. At time t the displacement of P from the line AB is x.

(a) Show that the tension in each string is $\frac{6mg}{l}\left(\sqrt{9l^2 + x^2} - l\right)$. [2]

(b) Show that

$$\ddot{x} = -\frac{12gx}{l}\left(1 - \frac{1}{\sqrt{9l^2 + x^2}}\right).$$ [4]

(c) Given that throughout the motion $\frac{x^2}{l^2}$ is small enough to be negligible, show that the equation of motion is approximately

$$\ddot{x} = -\frac{8gx}{l}.$$ [2]

(d) Given that the initial speed of P is $\sqrt{\frac{gl}{200}}$, find the time taken for the particle to travel a distance of $\frac{1}{80}l$. [4]

